Fermat‘s Proposition

Fermat (1601-1665)

Proof according to

Norbert Südland

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Statement

- **The equation $x^n + y^n = z^n$ cannot be solved for natural numbers x, y, z, and a natural number $n > 2$.**
- Power term: $z^n := n$ factors of z
- Natural number: integer number $z > 0$
- Each variable x, y, z, and n stands for a number, here of the natural numbers.

1^{st} Substitution of the Equation

- Switch to: $(x/y)^n = (z/y)^n - (y/y)^n = q^n - 1$
- Geometric sequence as telescope sum:
  $q^n - 1 = (q - 1)(q^{n-1} + \cdots + q^{\mu} + \cdots + q + 1)$
- $z^n - y^n = (z - y)(z^{n-1} + \cdots + z^{n-1-\mu} y^{\mu} + \cdots + y^{n-1})$
- Factor $(z - y)$ be b (is often 1) for $n > 1$.
- Special case $n = 1$ is soluble: $x + y = z$
- Thus 1^{st} substitution for $n > 1$: $z = y + b$

2^{nd} Substitution for $b = 1$

- Rest problem for $b = 1$: $x^n = (y + 1)^n - y^n$
- Special case $n = 2$: $x^2 = 2\,y + 1$
  leads to all odd square numbers.
- For $n > 2$ the Binomial theorem is forcing:
  $x^3 = (y + 1)^3 - y^3 = 3\,y^2 + 3\,y + 1$
  $x^4 = (y + 1)^4 - y^4 = 4\,y^3 + 6\,y^2 + 4\,y + 1$ etc.
- Thus 2^{nd} substitution for $n > 2$: $x^n = -\,y^n$

Pythagorean Twins

- Rest problem for $b > 1$: $x^n = (y + b)^n - y^n$
- Special case $n = 2$, fixed b: $x^2 = 2\, b\, y + b^2$ yields the missing Pythagorean twins:
- $y = (x^2 - b^2) / (2\, b)$ is often soluble, e.g.:
- $b = 2$: $17^2 - 15^2 = 8^2 = x^2$ (irreducible)
- $b = 3$: always reducible by division by 9
- $b = 9$: $149^2 - 140^2 = 51^2$ (irreducible)

2^{nd} Substitution for $b > 1$

- For $n > 2$ the Binomial theorem is forcing:
  $x^3 = (y + b)^3 - y^3 = \quad 3\,b\,y^2 + 3\,b^2\,y + b^3$
  $x^4 = (y + b)^4 - y^4 = 4\,b\,y^3 + 6\,b^2\,y^2 + 4\,b^3\,y + b^4$
  etc.
- For $n > 2$ the Binomial theorem (by Fermat andPascal) is fundamental in such a manner, that there is no way to go around.
- Thus 2^{nd} substitution for $n > 2$: $\quad x^n = - y^n$

Conclusion

- The equation $x^n = - y^n$ always leads out of the set of natural numbers.
- Therefore follows the statement:
  The equation $x^n + y^n = z^n$ cannot be solved for natural numbers x, y, z, and a natural number $n > 2$.
- quod erat demonstrandum (That‘s it).

Acknowledgement to the Following Person

- Professor Dr. Bodo Volkmann (Stuttgart): He named an irreducible example for $b > 1$ and $n = 2$.
- **Hint:**
  If you find a further weakness within this way to prove Fermat‘s theorem, then please contact the following person:
- Norbert.Suedland@t-online.de